AD-A115 821 FLORIDA STATE UNIV TALLAHASSEE DEPT OF STATISTICS F/G 12/1
RANKING AND SUBSET SELECTION PROCEDURES FOR EXPONENTIAL POPULAT--ETC(U)
MAY 82 R L BERGER, J S KIM DAAG29-79-C-0158
UNCLASSIFIED FSU-STATISTICS-M628 ARO-16713.18-MA NL

1 OF 1
AD A
115821

END
DATE
FILMED
07-82
DTIC

Ranking and Subset Selection Procedures for Exponential Populations with Type-I and Type-II Censored Data

by

Roger L. Berger[1] and Jee Soo Kim[2]

FSU Statistics Report M628
USARO Technical Report D52

The Florida State University
Department of Statistics
Tallahassee, Florida 32306

May, 1982

[1]This report was supported in part by the U.S. Army Research Office Grant No. DAAG29-79-C-0158. The United States Government is authorized to reproduce and distribute reprints for governmental purposes.

[2]Research performed in part with the support of the National Institute of Environmental Health Sciences, under Grant 5 T32 ES07011-05.

Ranking and Subset Selection Procedures for Exponential Populations with Type-I and Type-II Censored Data

by

Roger L. Berger and Jee Soo Kim

ABSTRACT

In this paper ranking and subset selection procedures for exponential populations with respect to the largest location and scale parameters are proposed. The data are assumed to be generated from Type-I and Type-II censoring mechanisms. The selection procedure proposed for the largest scale parameter based on Type-II censored data is equivalent to Gupta's procedure for gamma populations. Two procedures proposed for the selection of the largest location parameter under Type-I censoring and Type-II censoring are asymptotically equivalent.

1. INTRODUCTION

In this paper we investigate the problem of ranking and selection for exponential populations with Type-I and Type-II censored data. We consider both the indifference zone formulation of Bechhofer (1954) and the subset selection formulation of Gupta (1956). To our knowledge, W. Huang and K. Huang (1980) are the only researchers who have considered a selection problem with incomplete data.

Censored or incomplete data arise in various situations such as industrial life-testing, clinical trials, and biological experiments. To motivate our discussion and to illustrate how incomplete data can arise, we present a particular situation from industrial life-testing. A batch of n electronic components (or items) are placed on test at time 0, and the experiment terminates at a pre-specified time T. The failure time of a component is observable if it fails before time T. If a component still functions at the closure of the experiment, its failure time is not observable. The component is said to be censored at time T. This type of time censoring is known as Type-I censoring. In this life-testing experiment an experimenter might not know beforehand what value of the fixed censoring time T is appropriate, so he may decide to continue the experiment until a pre-specified number r, $1 \le r \le n$, (or a fraction $\frac{r}{n}$, or a proportion $100\alpha\%$, $0 < \alpha < 1$) of the components have failed. Those components still functioning at the time of the r^{th} failure are called censored observations. This type of failure censoring is known as Type-II censoring. Type-I and Type-II censoring schemes have received much attention in the statistical literature. See Bain (1978), Barlow and Proschan (1966, 1967), Epstein and Sobel (1953), and Mann et al. (1974).

We deal specifically with exponential distributions subject to these two types of censoring mechanisms. Location parameter problems are considered in Section 2 under Type-I censoring and in Section 3 under Type-II censoring. In Section 4, ranking and selection problems for scale parameters under Type-II censoring are discussed. Tables for implementing the new procedures are presented. Throughout this paper, we use the word increasing in place of nondecreasing and decreasing in place of nonincreasing.

2. SUBSET SELECTION FOR THE LARGEST LOCATION PARAMETER BASED ON TYPE-I CENSORED DATA

Let $\Pi_1, \Pi_2, \ldots, \Pi_k$ denote $k(k \geq 2)$ independent exponential populations with density functions

$$f(x; \lambda_i) = \begin{cases} \frac{1}{\theta} e^{-(x-\lambda_i)/\theta} & , x \geq \lambda_i \\ 0, x < \lambda_i, i = 1, 2, \ldots, k. \end{cases}$$

The scale parameter θ is assumed known. The location parameter λ_i are the unknown parameters of interest. The parameter space will be denoted by

$$\Lambda = \{\underline{\lambda}: \underline{\lambda} = (\lambda_1, \lambda_2, \ldots, \lambda_k), \lambda_i \geq 0\}.$$

Let $\lambda_{[1]} \leq \lambda_{[2]} \leq \ldots \leq \lambda_{[k]}$ denote the ordered values of $\lambda_1, \lambda_2, \ldots, \lambda_k$ and let $\Pi_{(i)}$ denote the unknown population associated with $\lambda_{[i]}$. Our goal is the subset selection goal formulated by Gupta (1956). This goal is to select a nonempty subset of the k populations containing $\Pi_{(k)}$, the population associated with the largest parameter $\lambda_{[k]}$. A correct selection, denoted CS, is the selection of any subset which contains $\Pi_{(k)}$. The population $\Pi_{(k)}$ is referred to as the best population. If more than one population could be classified as best, then one of these is arbitrarily tagged as the best. This is done only for the purpose of evaluating the infimum of the probability of a CS. We shall propose a selection rule R_1 which satisfies tho P^*-condition,

$$\inf_{\Lambda} P_{\underline{\lambda}}(CS|R_1) \geq P^*, \tag{2.1}$$

where $\frac{1}{k} < P^* < 1$ is specified by the experimenter prior to experimentation.

For $i = 1, 2, \ldots, k$ let X_{ij}, $j = 1, 2, \ldots, n$ denote a random sample of size n from Π_i. The X_{ij} might be the life lengths of n items from population Π_i placed on test at time 0. The X_{ij} are not themselves observable. Rather they are subjected to Type-I censoring. That is, there is a specified time $T > 0$ past which no observation will take place. Thus the observable data are X^*_{ij}, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, n$, where $X^*_{ij} = \min(X_{ij}, T)$. Using the Factorization Theorem (Lehmann (1959), P. 49) it can be shown that $(Y_1, Y_2, \ldots, Y_k)$ is a sufficient statistic in this problem where

$$Y_i = \min(X^*_{i1}, \ldots, X^*_{in}) = \min(\min(X_{i1}, \ldots, X_{in}), T).$$

The selection procedure we propose depends on the data only through the sufficient statistic $(Y_1, Y_2, \ldots, Y_k)$.

The observations $Y_1, Y_2, \ldots, Y_k$ are independent random variables. The c.d.f. of Y_i is $G(y_i; \lambda_i)$ where

$$G(y; \lambda) = \begin{cases} 0 & \text{if } y < \lambda \\ 1 - e^{-n(y-\lambda)/\theta} & \text{if } \lambda \leq y < T \\ 1 & \text{if } T \leq y \end{cases}$$

if $\lambda < T$ and

$$G(y; \lambda) = \begin{cases} 0 & \text{if } y < T \\ 1 & \text{if } T \leq y \end{cases}$$

if $T \le \lambda$. If $\lambda < T$, $G(y;\lambda)$ is a mixed probability distribution with an absolutely continuous part defined by the density $g(y;\lambda) = \frac{n}{\theta} e^{-n(y-\lambda)/\theta}$ for $\lambda \le y < T$ and a discrete part consisting of a point mass at T with probability $e^{-n(T-\lambda)/\theta}$. If $T \le \lambda$, $G(y;\lambda)$ is degenerate at T.

The selection procedure we propose is denoted by R_1.

<u>Procedure R_1</u>: Select Π_i if and only if

$$Y_i \ge \max_{1\le j\le k} Y_j - \frac{d\theta}{n} \tag{2.2}$$

where $d = d(k, P^*, \theta, T, n) \ge 0$ is chosen so the P^*-condition (2.1) is satisfied.

The method of computing d will be given. It will be shown that d depends on n, T and θ only through $\frac{nT}{\theta}$ and that $0 \le d \le \frac{nT}{\theta}$. Table 1 gives values of d for $k = 2(1)5$, $\frac{nT}{\theta} = .25(.25)4.00$ and $5(1)10$ and $P^* = .75(.05).95$ and $.99$.

If $d \ge \frac{nT}{\theta}$ then $\max_{1\le j\le k} Y_j - \frac{d\theta}{n} \le 0$ with probability 1 for any $\underline{\lambda}$. Thus $P_{\underline{\lambda}}(CS|R_1) = 1$ for all $\underline{\lambda}$. For $P^* < 1$, (2.1) can be achieved with a $d < \frac{nT}{\theta}$.

We will now examine $P_{\underline{\lambda}}(CS|R_1)$ to determine how d should be chosen to ensure the P^*-condition (2.1).

<u>Theorem 2.1</u>. Let $\bar{d} = \frac{d\theta}{n}$. Then

$$P_{\underline{\lambda}}(CS|R_1) = \begin{cases} 1 & \text{for } T - \bar{d} \le \lambda_{[k]} \\ \int_{\lambda_{[k]}}^{T-\bar{d}} \{\prod_{j=1}^{k-1} [1-e^{-n(y+\bar{d}-\lambda_{[j]})/\theta}]\} \frac{n}{\theta} e^{-n(y-\lambda_{[k]})/\theta} dy & \\ + e^{-n(T-\bar{d}-\lambda_{[k]})/\theta} & \text{for } \lambda_{[k]} < T - \bar{d}. \end{cases} \tag{2.3}$$

<u>Proof</u>. Let $Y_{(j)}$, $j = 1, 2, \ldots, k$ be the observation from $\Pi_{(j)}$.

If $T - \bar{d} \le \lambda_{[k]}$, $1 = P_{\underline{\lambda}}(Y_{(k)} \ge T - \bar{d}) \le P_{\underline{\lambda}}(Y_{(k)} \ge \max\limits_{1 \le j \le k} Y_{(j)} - \bar{d})$ $= P_{\underline{\lambda}}(CS|R_1) \le 1$.

If $\lambda_{[k]} < T - \bar{d}$,

$$\begin{aligned}
P_{\underline{\lambda}}(CS|R_1) &= P_{\underline{\lambda}}(Y_{(k)} \ge Y_{(j)} - \bar{d},\ j=1, 2, \ldots, k-1, \text{ and } Y_{(k)} < T - \bar{d}) \\
&\quad + P_{\underline{\lambda}}(Y_{(k)} \ge Y_{(j)} - \bar{d},\ j=1, 2, \ldots, k-1, \text{ and } Y_{(k)} \ge T - \bar{d}) \\
&= P_{\underline{\lambda}}(Y_{(k)} + \bar{d} \ge Y_{(j)},\ j=1, 2, \ldots, k-1, \text{ and } Y_{(k)} < T - \bar{d}) \\
&\quad + P_{\underline{\lambda}}(Y_{(k)} \ge T - \bar{d}) \\
&= \int_{\lambda_{[k]}}^{T-\bar{d}} \prod_{j=1}^{k-1} G(y + \bar{d};\ \lambda_{[j]}) g(y;\ \lambda_{[k]}) dy \\
&\quad + e^{-(T-\bar{d}-\lambda_{[k]})/\theta}
\end{aligned}$$

Substituting for G and g yields (2.3). ||

For a fixed value of $\lambda_{[k]}$, the $P_{\underline{\lambda}}(CS|R_1)$ in (2.3) decreases if $\lambda_{[1]}, \lambda_{[2]}, \ldots, \lambda_{[k]}$ are all replaced by $\lambda_{[k]}$. Thus the $\inf\limits_{\Lambda} P_{\underline{\lambda}}(CS|R_1)$ takes place when $\lambda_{[1]} = \lambda_{[2]} = \ldots = \lambda_{[k]}$. If $\underline{\lambda} = (\lambda, \lambda, \ldots, \lambda)$ and $\lambda < T - \bar{d}$, then

$$\begin{aligned}
P_{\underline{\lambda}}(CS|R_1) &= \int_{\lambda}^{T-\bar{d}} \{1 - e^{-n(y+\bar{d}-\lambda)/\theta}\}^{k-1} \frac{n}{\theta} e^{-n(y-\lambda)/\theta} dy \\
&\quad + e^{-n(T-\bar{d}-\lambda)/\theta} \\
&= \frac{1}{k} e^{d} \{1 - e^{-n(y+\bar{d}-\lambda)/\theta}\}^{k} \Big|_{\lambda}^{T-\bar{d}} + e^{-n(T-\bar{d}-\lambda)/\theta} \\
&= \frac{1}{k} e^{d} \{1 - e^{-n(T-\lambda)/\theta}\}^{k} - \frac{1}{k} e^{d} \{1 - e^{-d}\}^{k} + e^{-n(T-\bar{d}-\lambda)/\theta}.
\end{aligned}$$

Differentiation with respect to λ shows that $P_{\underline{\lambda}}(CS|R_1)$ is a monotone increasing function of λ for $0 \le \lambda < T - \bar{d}$.

Therefore, $P_{\underline{\lambda}}(CS|R_1)$ is minimized by setting $\lambda = 0$.

Theorem 2.2. For any value of d, $0 \le d \le \frac{nT}{\theta}$,

$$\inf_{\Lambda} P_{\underline{\lambda}}(CS|R_1) = e^d\{\frac{1}{k}(1 - e^{-\frac{nT}{\theta}})^k - \frac{1}{k}(1 - e^{-d})^k + e^{-\frac{nT}{\theta}}\}. \tag{2.4}$$

We have restricted d to be nonnegative so that a nonempty subset is always selected. If $d = 0$, then from (2.4) we see that $\inf_{\Lambda} P_{\underline{\lambda}}(CS|R_1) = \frac{1}{k}(1 - e)^{-\frac{nT}{\theta}})^k + e^{\frac{-nT}{\theta}}$. If P^* is less than this value, then $d = 0$ can be used and strict inequality holds in (2.1). This can be observed for some small P^* values in Table 1. But if P^* is greater than or equal to this value, d can be chosen so that equality holds in (2.1) by equating the right side of (2.4) to P^* and solving for d. Note that (2.4) depends on n, θ, and T only through $\frac{nT}{\theta}$. Thus we have Theorem 2.3.

Theorem 2.3. For $k \ge 2$, $\frac{nT}{\theta} > 0$ and $\frac{1}{k}(1 - e^{-\frac{nT}{\theta}})^k + e^{-\frac{nT}{\theta}} \le P^* < 1$, if $d = d(k, P^*, \frac{nT}{\theta}) = -\ln u$ where u is the solution of the polynomial

$$(1 - u)^k + kP^*u - ke^{-\frac{nT}{\theta}} - (1 - e^{-\frac{nT}{\theta}})^k = 0 \tag{2.5}$$

in the interval $e^{-\frac{nT}{\theta}} < u \le 1$, then the P^*-condition (2.1) is satisfied with equality.

The values of d in Table 1 were computed by solving (2.5). Let $\underline{0} = (0, 0, \ldots, 0)$. Since $P_{\underline{0}}(CS|R_1)$(the expression in (2.4)) is a continuous strictly increasing function of d for $0 \le d \le \frac{nT}{\theta}$ and $P_{\underline{0}}(CS|R_1) = 1$ if $d = \frac{nT}{\theta}$, (2.5) always has a unique solution in $e^{-\frac{nT}{\theta}} < u \le 1$. As $\frac{nT}{\theta} \to \infty$, $d(k, P^*, \frac{nT}{\theta}) \to d(k, P^*, \infty)$ where $d(k, P^*, \infty)$ is the solution of

$$e^d\{\frac{1}{k} - \frac{1}{k}(1 - e^{-d})^k\} = P^*, \quad d \ge 0. \tag{2.6}$$

Equivalently $d(k, P^*, \infty) = -\ln u$ where u is the solution of

$$(1 - u)^k + kP^*u - 1 = 0, \quad 0 < u \le 1.$$

The values of $d(k, P^*, \infty)$ are also listed in Table 1. For the values of k and P^* in Table 1,

$$|d(k, P^*, \infty) - d(k, P^*, 10)| \le .0001.$$

Finally, we note some reasonable properties of R_1 in Theorem 2.4.

<u>Theorem 2.4</u>. Fix $T > 0$ and $\theta > 0$.

(i) If $T \le \lambda_{[k]}$ or $\lambda_{[k-1]} < \lambda_{[k]} < T$, $\lim_{n\to\infty} P_{\underline{\lambda}}(CS|R_1) = 1$.

(ii) (Monotonicity) If $\lambda_i \le \lambda_j$, $P_{\underline{\lambda}}(\text{select } \Pi_i|R_1) \le P_{\underline{\lambda}}(\text{select } \Pi_j|R_1)$.

(iii) If $\lambda_{[k-1]} < \text{minimum}(\lambda_{[k]}, T)$, $\lim_{n\to\infty} E_{\underline{\lambda}}(S|R_1) = 1$

where S is the size of the selected subset.

Proof.

(i) If $T \le \lambda_{[k]}$, $P_{\underline{\lambda}}(CS|R_1) = 1$ for every n. Suppose $\lambda_{[k-1]} < \lambda_{[k]} < T$. Since $Y_{(i)} \to \lambda_{[i]}$ a.s. as $n \to \infty$,

$P_{\underline{\lambda}}(CS|R_1) \ge P_{\underline{\lambda}}(Y_{(k)} = \max_{1 \le j \le k} Y_{(j)}) \to 1$ as $n \to \infty$.

(ii) $G(y; \lambda)$ is stochastically increasing in λ, i.e., $G(y; \lambda_j) \le G(y; \lambda_i)$ if $\lambda_i \le \lambda_j$. This fact and Problem 11, page 112 of Lehmann (1959) yield

$$P_{\underline{\lambda}}(\text{select } \Pi_i|R_1) = \int_0^T \prod_{\substack{m=1\\ m\ne i}}^{k} G(y + \frac{d\theta}{n} ; \lambda_m) dG(y; \lambda_i)$$

$$\le \int_0^T \prod_{\substack{m=1\\ m\ne j}}^{k} G(y + \frac{d\theta}{n} ; \lambda_m) dG(y; \lambda_i)$$

$$\le \int_0^T \prod_{\substack{m=1\\ m\ne j}}^{k} G(y + \frac{d\theta}{n} ; \lambda_m) dG(y; \lambda_j)$$

$$= P_{\underline{\lambda}}(\text{select } \Pi_j|R_1).$$

(iii) $E_{\underline{\lambda}}(S|R_1) = \sum_{i=1}^{k} P_{\underline{\lambda}}(\text{select } \Pi_{(i)}|R_1)$.

By (i) and (ii) it suffices to show $\lim_{n\to\infty} P_{\underline{\lambda}}(\text{select } \Pi_{(k-1)}|R_1) = 0$.

Since $\lim_{n\to\infty} d = \lim_{n\to\infty} d(k, P^*, \frac{nT}{\theta}) = d(k, P^*, \infty) < \infty$, $\lim_{n\to\infty} \frac{d\theta}{n} = 0$.

$$P_{\underline{\lambda}}(\text{select } \Pi_{(k-1)}|R_1) \le 1 - P_{\underline{\lambda}}(Y_{(k-1)} < Y_{(k)} - \frac{d\theta}{n})$$

$$\le 1 - P_{\underline{\lambda}}(Y_{(k-1)} < \text{minimum}(T, \lambda_{[k]}) - \frac{d\theta}{n}).$$

But $\lim_{n\to\infty} P_{\underline{\lambda}}(Y_{(k-1)} < \text{minimum}(T, \lambda_{[k]}) - \frac{d\theta}{n}) = 1$, since $Y_{(k-1)} \to \lambda_{[k-1]} < \text{minimum}(T, \lambda_{[k]})$ a.s. as $n \to \infty$ and $\frac{d\theta}{n} \to 0$ as $n \to \infty$. ||

If T is relatively small, then $E_{\underline{\lambda}}(S|R_1)$ will be relatively large, even for large n. More precisely, if $\lambda_{[i]} < T \le \lambda_{[i+1]}$ then $\lim_{n\to\infty} E_{\underline{\lambda}}(S|R_1) = k - i$. Thus setting T small may result in savings of experiment time, but may also result in the selection of a large subset.

3. SUBSET SELECTION FOR THE LARGEST LOCATION PARAMETER BASED ON TYPE-II CENSORED DATA

Assume that for $i = 1, 2, \ldots, k$, X_{ij}, $j = 1, 2, \ldots, n$ denote a random sample of size n from Π_i. Thus the X_{ij} are defined as in Section 2. Again, we are interested in selecting a subset containing $\lambda_{[k]}$. In this section, we consider Type-II censored data, rather than the Type-I censored data of Section 2.

Let r_i, $i = 1, \ldots, k$ be fixed integers satisfying $1 \leq r_i \leq n$. Let $X_{i[1]} \leq X_{i[2]} \leq \cdots \leq X_{i[n]}$ denote the order statistics from population Π_i. The data from population Π_i is Type-II censored if the observations available from Π_i are $X_{i[1]}, \ldots, X_{i[r_i]}$. This type of data arise if n items are placed on test simultaneously and observation stops after the r_i^{th} failure.

Assume that Type-II censorod data are available from each Π_i, $i = 1, 2, \ldots, k$, i.e., the data are $X_{i[j]}$, $i = 1, 2, \ldots, k$, $j = 1, 2, \ldots, r_i$. Using the Factorization Theorem (Lehmann, 1959), it can be shown that $(X_{1[1]}, X_{2[1]}, \ldots, X_{k[1]})$ is a sufficient statistic for this problem. Thus the experimenter needs only observe the first failure time from each population. Alternatively, $(X_{1[1]}, X_{2[1]}, \ldots, X_{k[1]})$ may be thought of as the result of Type-II censoring with $r_i = 1$, $i = 1, \ldots, k$.

Ofosu (1974) proposed the following subset selection rule for $\lambda_{[k]}$ based on $(X_{1[1]}; X_{2[1]}, \ldots, X_{k[1]})$.

<u>Procedure R_2</u>: Select Π_i if and only if

$$X_{i[1]} \geq \max_{1\leq j\leq k} X_{j[1]} - \frac{d\theta}{n}, \quad i = 1, 2, \ldots, k$$

where $d = d(k, P^*) \geq 0$ satisfies

$$\frac{1}{k} e^{d}(1 - (1 - e^{-d})^{k}) = P^* \tag{3.1}$$

Ofosu showed that for $\frac{1}{k} < P^* < 1$, R_2 satisfies the P^*-condition

$$\inf_{\Lambda} P_{\underline{\lambda}}(CS|R_2) \geq P^*$$

if d is chosen as in (3.1). Condition (3.1) is the limiting case (2.6) for the procedure R_1 discussed in Section 2. This is unsurprising since $P_{\underline{\lambda}}(Y_i = X_{i[1]}) \to 1$ as $T \to \infty$ for fixed n and θ.

Some desirable properties of the rule R_2 are listed below. Properties (i) and (ii) were noted by Ofosu (1974). Properties (iii) and (iv) can be proven in a similar fashion to Theorem 2.4.

(i) $\sup_{\Lambda} E_{\underline{\lambda}}(S|R_2) = kP^*$.

(ii) If $\lambda_i \leq \lambda_j$, $P_{\underline{\lambda}}(\text{select } \Pi_i|R_2) \leq P_{\underline{\lambda}}(\text{select } \Pi_j|R_2)$.

(iii) If $\lambda_{[k-1]} < \lambda_{[k]}$, $\lim_{n\to\infty} P_{\underline{\lambda}}(CS|R_2) = 1$.

(iv) If $\lambda_{[k-1]} < \lambda_{[k]}$, $\lim_{n\to\infty} E_{\underline{\lambda}}(S|R_2) = 1$.

Property (i) is in marked contrast to R_1 in the Type-I censoring model. If $\lambda_{[1]} \geq T$, $E_{\underline{\lambda}}(S|R_1) = k$ (all the populations are surely selected). Berger (1979) showed that if a selection rule satisfies (i) then the rule is minimax with respect to S and S′, the number of non-best populations selected. That is, R_2 satisfies

$$\sup_{\Lambda} E_{\underline{\lambda}}(S|R_2) = \inf_{\mathcal{D}_{P^*}} \sup_{\Lambda} E_{\underline{\lambda}}(S|R)$$

and

$$\sup_{\Lambda} E_{\underline{\lambda}}(S'|R_2) = \inf_{\mathcal{D}_{P^*}} \sup_{\Lambda} E_{\underline{\lambda}}(S'|R)$$

where $\mathcal{D}_{P^*}$ is the set of all rules which satisfy the P^*-condition. In the Type-I censoring model of Section 2, no nonrandomized rule can be minimax since $P_{\underline{\lambda}}(Y_i = T,\ i = 1, 2, \ldots, k) = 1$ if $\lambda_{[1]} \geq T$.

4. RANKING AND SUBSET SELECTION FOR SCALE PARAMETERS BASED ON TYPE-II CENSORED DATA.

Let $\Pi_1, \Pi_2, \ldots, \Pi_k$ be $k (k \geq 2)$ exponential populations with density functions

$$f(x; \theta_i) = \begin{cases} \frac{1}{\theta_i} e^{-x/\theta_i}, & \text{if } x > 0 \text{ and } \theta_i > 0 \\ 0 & \text{otherwise}, \quad i = 1, 2, \ldots, k. \end{cases} \tag{4.1}$$

The ordered values of the unknown scale parameter θ_i are denoted by

$$\theta_{[1]} \leq \theta_{[2]} \leq \ldots \leq \theta_{[k]}. \tag{4.2}$$

The unknown population associated with $\theta_{[i]}$ is denoted by $\Pi_{(i)}$. It is assumed that there is no prior knowledge about the correct pairing of the k populations and the ordered scale parameters $\theta_{[i]}$. In the context of the reliability theory and life-testing models θ_i is the mean life of an item whose life length is described by an exponential distribution function. The population with the parameter θ_i equal to $\theta_{[k]}$ is called the best population. The goal is to identify the best population, and also the $t(1 \leq t \leq k - 1)$ best populations.

From each population Π_i, $i = 1, 2, \ldots, k$ we take a sample of n items. Let $X_{i[1]} \leq X_{i[2]} \leq \cdots \leq X_{i[n]}$ denote the order statistics representing the failure times of the n items from population Π_i. We consider Type-II censoring as in Section 3. Let r be a fixed integer such that $1 \leq r \leq n$. Under Type-II censoring the first r failures from each population Π_i are to be observed. Observation on Π_i ceases with the observation of $X_{i[r]}$. The (n - r) items whose failure times are not observable beyond the $X_{i[r]}$ become censored observations. Thus, the length of the time population Π_i is observed is the random variable $X_{i[r]}$. Type-II censoring was first investigated by Epstein and Sobel (1953).

Section 4.1 introduces a statistic known as the <u>total time on test</u> (TTOT) statistic generated from Type-II censoring. Some of its well-known properties are stated in Section 4.1. Selection rules are proposed on the basis of the TTOT statistics.

<u>4.1. Total Time on Test (TTOT) Statistics and Its Properties</u>.

For the Type-II censored sampling the likelihood function of the first r failures, $X_{i[1]}, X_{i[2]}, \ldots, X_{i[r]}$ from Π_i is given by

$$f(x_{i[1]},x_{i[2]},\ldots,x_{i[r]};\theta_i) = \begin{cases} \dfrac{n!}{(n-r)!\,\theta_i^r} \exp[-\dfrac{1}{\theta_i}(\sum_{j=1}^{r} x_{i[j]}+(n-r)x_{i[r]})] \\ \text{for } 0 \le x_{i[1]} \le x_{i[2]} \le \ldots \le x_{i[r]} < \infty \\ 0, \text{ otherwise.} \end{cases} \tag{4.3}$$

The maximum likelihood estimate (MLE) $\hat{\theta}_i$ of θ_i obtained by setting $\frac{d \log f}{d\theta_i} = 0$ is

$$\hat{\theta}_i = \frac{1}{r} Z_i \tag{4.4}$$

where $Z_i = \sum_{j=1}^{r} X_{i[j]} + (n - r)X_{i[r]}$, which can be written as

$$Z_i = \sum_{j=1}^{r} (n - j + 1)(X_{i[j]} - X_{i[j-1]}) + (n - r)X_{i[r]}, \qquad (4.5)$$

where $X_{i[0]} \equiv 0$. Z_i represents the total time spent on observing n items undergoing life-testing until the r^{th} failure from Π_i whether an item is censored or uncensored. Z_i is called the TTOT statistic. We state some of the known properties of $\hat{\theta}_i$. See, for example, Epstein and Sobel (1953), Mann et at. (1974) and Bain (1978).

Lemma 4.4.1. $\hat{\theta}_i$ defined in (4.4) has the following properties.

(i) $\hat{\theta}_i$ is a complete and sufficient statistic for θ_i.

(ii) $\hat{\theta}_i$ is the MLE of θ_i.

(iii) $\hat{\theta}_i$ is the UMVUE of θ_i.

(iv) $\hat{\theta}_i$ is a strongly consistent estimator of θ_i, if $\frac{r}{n}$ is fixed.

(v) $\frac{2r\hat{\theta}_i}{\theta_i}$ is distributed as $\chi^2_{(2r)}$, a chi-square with 2r degrees of freedom.

(vi) $Z_i = r\hat{\theta}_i$ has a gamma distribution with shape parameter r and scale parameter θ_i.

Remark 4.1.1. The distribution of Z_i is independent of the sample size n. The advantage of r failures out of a sample of n items

$(n > r)$ over r failures out of a sample of r items is in the savings of time. The expected waiting time to observe the r^{th} failure in the sample of n, $E(X_{i[r]}) = \theta_i \sum_{j=1}^{r} \frac{1}{n-j+1}$, is less than the expected waiting time for all r failures in the sample of r items.

Procedures based on Z_i, equivalently on $\hat{\theta}_i$, are proposed with both formulations, indifference zone formulation and subset selection formulation, in mind.

4.2. Indifference Zone Formulation.

Our goal is to select t, $1 \le t \le k - 1$, populations associated with the t largest scale parameters $\theta_{[\beta]}$, $\beta = k - t + 1, k - t + 2, \ldots, k$ whenever the unknown parameter configuration $\underline{\theta} = (\theta_1, \theta_2, \ldots, \theta_k)$ is assumed to lie outside some subset of the entire parameter space, denoted by Θ. The selection of the subset containing exactly the t largest populations will be called the correct selection (CS). The preference zone, $\Theta(\delta)$ is defined by

$$\Theta(\delta) = \{\underline{\theta} \in \Theta: \ \theta_{[k-t]}/\theta_{[k-t+1]} < \delta\} \tag{4.6}$$

where $0 < \delta < 1$ is fixed by the experimenter.

We shall propose a selection procedure R_3 for which the probability of the correct selection satisfies the following inequality

$$P_{\underline{\theta}}(CS|R_3) \ge P^*, \ \frac{1}{\binom{k}{t}} < P^* < 1 \text{ for all } \underline{\theta} \in \Theta(\delta), \tag{4.7}$$

where δ and P^* are specified in advance. This is the formulation of the selection problem presented by Bechhofer (1954). Let the ordered values of Z_i be denoted by

$$Z_{[1]} \leq Z_{[2]} \leq \ldots \leq Z_{[k]} \tag{4.8}$$

The procedure R_3 is defined as follows.

Procedure R_3: Select t populations corresponding to $Z_{[k]}$, $Z_{[k-1]}, \ldots, Z_{[k-t+1]}$ (i.e. select population Π_i for which $Z_i \geq Z_{[k-t+1]}$). (4.9)

The rule R_3 which depends on observations (censored or uncensored) only through the TTOT statistics Z_i is equivalent to the procedure based on the MLE's $\hat{\theta}_i$.

Having specified two quantities δ and P^* an experimenter is now interested in determining the smallest value of r to guarantee the basic probability requirement (4.7). For this purpose the infimum of $P_{\underline{\theta}}(CS|R_3)$ must be obtained. Let the TTOT statistic from the population associated with $\theta_{[i]}$ be denoted by $Z_{(i)}$. It can be shown that

$$P_{\underline{\theta}}(CS|R_3) = P_{\underline{\theta}}(\max\{Z_{(1)},\ldots,Z_{(k-t)}\} < \min\{Z_{(k-t+1)},\ldots,Z_{(k)}\}) \tag{4.10}$$

and that the least favorable configuration in $\Theta(\delta)$ is given by $\theta_{[1]} = \theta_{[2]} = \cdots = \theta_{[k-t]}$, $\theta_{[k-t]} = \delta\theta_{[k-t+1]}$, and $\theta_{[k-t+1]} = \cdots = \theta_{[k]}$. Thus, we can obtain

$$\inf_{\Theta(\delta)} P_{\underline{\theta}}(CS|R_3) = (k-t)\int_0^\infty F_r^{k-t-1}(u)(1 - F_r(\delta u))^t dF_r(u), \tag{4.11}$$

where $F_r(u) = \frac{1}{\Gamma(r)} \int_0^u x^{r-1} e^{-x} dx$.

Since $Z_1, \ldots, Z_k$ are independent gamma variables with common known shape parameter r and unknown scale parameter θ_i, the selection rule R_3 is closely related to the selection rule studied by Bechhofer and Sobel (1954) in their investigation of the ranking variances of normal populations based on the sample variances. Tables in Bechhofer and Sobel (1954) or Gibbons, Olkin and Sobel (1977) can be used to determine the minimum r required to satisfy (4.7).

We now study the behaviour of the $P_{\underline{\theta}}(CS|R_3)$ *as* δ *and* n *vary.*

Theorem 4.2.1. For any fixed r, $1 \le r \le n$,

$$\lim_{\delta \to 0} \inf_{\Theta(\delta)} P_{\underline{\theta}}(CS|R_3) = 1.$$

Proof. This follows readily from the expression (4.11),

$$\begin{aligned} \inf_{\Theta(\delta)} P_{\underline{\theta}}(CS|R_3) &= (k - t) \int_0^\infty F_r^{k-t-1}(u) \ (1 - F_r(\delta u))^t dF_r(u) \\ &\to (k - t) \int_0^\infty F_r^{k-t-1}(u) dF_r(u) \text{ as } \delta \to 0 \\ &= 1. \ \| \end{aligned}$$

Suppose that the experimenter specifies a fraction $\frac{r}{n}$, say $q(0 < q < 1)$, instead of fixing r. Then, as a consequence of Lemma 4.1.1 (iv) we get:

<u>Theorem 4.2.2</u>. Let $\frac{r}{n} = q$ be a fixed fraction. Then for any $\underline{\theta} \in \Theta(\delta)$, $\lim_{n\to\infty} P_{\underline{\theta}}(CS|R_3) = 1$.

4.3 Subset Selection Formulation

The approach we take in the present section is to select a subset of k given populations which contain the population associated with the largest scale parameter $\theta_{[k]}$. A correct selection (CS) is defined as the selection of any subset which contains the best population. We require that the probability of including the best population in the subset selection to be at least a pre-assigned fixed number P^*, that is,

$$\inf_{\Theta} P_{\underline{\theta}}(CS|R) \geq P^*, \text{ where } \frac{1}{k} < P^* < 1. \tag{4.12}$$

4.3.1 Selection Procedure R_4 and Its Properties.

Based on the TTOT statistics $Z_i = \sum_{j=1}^{r} X_{i[j]} + (n - r)X_{i[r]}$, $i = 1, 2, \ldots, k$, W. Huang and K. Huang (1980) proposed the following procedure.

<u>Procedure R_4</u>: Select the population Π_i if and only if

$$Z_i \geq c \max_{1\leq j\leq k} Z_j \tag{4.13}$$

where c is a positive constant, $0 < c < 1$, pre-assigned so as to satisfy the P^*-condition (4.12).

Since Z_i has a gamma distribution with the known shape parameter r and the scale parameter θ_i (Lemma 4.1.1 (vi)), the expression for the $\inf_{\Theta} P_{\underline{\theta}}(CS|R_4)$ and all the desirable properties of the rule R_4 are precisely the same as those of the rule proposed by Gupta (1963). Gupta (1963) considered a problem of a selection and ranking procedure for k gamma populations with respect to the largest scale parameter when the shape parameter are assumed to be known and all equal. He derived

$$\inf_{\Theta} P_{\underline{\theta}}(CS|R_4) = \int_0^\infty F_r^{k-1}(\frac{u}{c})dF_r(u) \tag{4.14}$$

where $F_r(u) = \frac{1}{\Gamma(r)} \int_0^u x^{r-1}e^{-x}dx$.

We note that the right-hand side of (4.14) is independent of the sample size n. It depends only on r, the number of observed failure times. For various values of k, r and P^*, the associated constants c satisfying the P^*-condition are tabulated in Gupta (1963).

As a possible question in designing experiments one may ask: For a fixed value of the constant c, $0 < c < 1$, what should the amount of the observed failures in the data be so as to satisfy the pre-assigned probability P^* specified by the experimenter? Monotonicity of the $\inf_{\Theta} P_{\underline{\theta}}(CS|R_4)$ will resolve this problem and will give us a unique smallest integer r for which $\inf_{\Theta} P_{\underline{\theta}}(CS|R_4) \geq P^*$.

Theorem 4.3.1. For any given values of k, c and P^* in the selection rule R_4

$$\inf_{\Theta} P_{\underline{\theta}}(CS|R_4) = \int_0^\infty F_r^{k-1}(\frac{u}{c})dF_r(u)$$

where $F_r(u) = \frac{1}{\Gamma(r)} \int_0^u t^{r-1}e^{-t}dt$, is strictly increasing in $r > 1$.

Proof. Suppose that X is a random variable having the density function $g_r(x) = c^r \exp[(1 - c)F_r^{-1}(x)]$ for $x \in [0, 1]$ and $r > 1$ where $F_r^{-1}(x)$ is the inverse function of the incomplete gamma, $F_r(u) = \frac{1}{\Gamma(r)} \int_0^u y^{r-1} e^{-y} dy$. Put $B(x) = \log g_{r_2}(x) - \log g_{r_1}(x)$ for $r_2 > r_1 > 1$ and $0 \le x \le 1$.

$\frac{dB(x)}{dx} = \frac{(1-c)\ A(x)}{f_{r_1}[F_{r_1}^{-1}(x)] f_{r_2}[F_{r_2}^{-1}(x)]}$ where $f_r(x)$ is a gamma density

and $A(x) = f_{r_1}[F_{r_1}^{-1}(x)] - f_{r_2}[F_{r_2}^{-1}(x)]$. Alam (1970) shows that $A(x)$ is nonnegative for all x, $x \in [0, 1]$. This proves $B(x)$ is increasing in x for all c, which in turn implies $g_r(x)$ is a density function having a monotone likelihood ratio in x. Hence, $E_r(X^{k-1}) = \int_0^1 x^{k-1} g_r(x)dx$ is increasing in r by Lehmann's lemma (1959, page 74). But

$$E_r(X^{k-1}) = \int_0^\infty F_r^{k-1}(\tfrac{u}{c})dF_r(u). \|$$

Suppose that the experimenter observes a fraction $\frac{r}{n}$ of failures. Then the procedure R_4 is a consistent procedure in a sense that for a sufficiently large n, the probability of including the best population in the subset selected approaches 1.

Theorem 4.3.2. Let $\frac{r}{n} = q$, $0 < q < 1$, be fixed. If $\theta_{[k-1]} < \theta_{[k]}$, then $\lim_{n \to \infty} P_{\underline{\theta}}(CS|R_4) = 1$.

Proof. This is immediate from Lemma 4.1.1 (iv), because R_4 is based on Z_i, or equivalently on $\hat{\theta}_i$ defined in (4.4). $\|$

Table 1. This table gives the necessary d-value required for the procedure R_4.

$P^* = .75$

$\frac{nT}{\theta}$ \ k	2	3	4	5	∞[5]
.25	.0000[1]	.0000[2]	.0000[3]	.0000[4]	.0000
.50	.0987	.1818	.2030	.2093	.2123
.75	.2428	.3847	.4297	.4477	.4623
1.00	.3642	.5681	.6405	.6736	.7123
1.25	.4616	.7309	.8342	.8851	.9623
1.50	.5360	.8711	1.0090	1.0805	1.2123
1.75	.5898	.9872	1.1628	1.2577	1.4623
2.00	.6268	1.0789	1.2934	1.4144	1.7123
2.25	.6514	1.1478	1.3996	1.5482	1.9623
2.50	.6672	1.1970	1.4820	1.6578	2.2123
2.75	.6772	1.2305	1.5426	1.7433	2.4623
3.00	.6834	1.2526	1.5851	1.8068	2.7123
3.25	.6872	1.2669	1.6137	1.8516	2.9623
3.50	.6895	1.2758	1.6324	1.8820	3.2123
3.75	.6909	1.2813	1.6442	1.9018	3.4623
4.00	.6918	1.2847	1.6517	1.9145	3.7123
5.00	.6930	1.2893	1.6619	1.9324	4.7123
6.00	.6931	1.2900	1.6633	1.9349	5.7123
7.00	.6931	1.2900	1.6635	1.9352	6.7123
8.00	.6931	1.2901	1.6636	1.9353	7.7123
9.00	.6931	1.2901	1.6636	1.9353	8.7123
10.00	.6931	1.2901	1.6636	1.9153	9.7123
∞	.6931	1.2901	1.6636	1.9353	∞

[1] Actual inf $P(CS|R_1) = .8033$

[2] Actual inf $P(CS|R_1) = .7824$

[3] Actual inf $P(CS|R_1) = .7794$

[4] Actual inf $P(CS|R_1) = .7789$

[5] $d \to \frac{nT}{\theta} + \ln P^*$ as $k \to \infty$

$P^* = 80$

$\frac{nT}{\theta}$ \ k	2	3	4	5	∞
.25	.0000[1]	.0222	.0261	.0267	.0269
.50	.1760	.2497	.2683	.2740	.2769
.75	.3383	.4610	.4988	.5141	.5269
1.00	.4801	.6566	.7168	.7442	.7769
1.25	.5989	.8350	.9210	.9629	1.0269
1.50	.6939	.9938	1.1098	1.1687	1.2769
1.75	.7658	1.1305	1.2806	1.3595	1.5269
2.00	.8176	1.2435	1.4309	1.5329	1.7769
2.25	.8531	1.3325	1.5584	1.6862	2.0269
2.50	.8766	1.3989	1.6617	1.8169	2.2769
2.75	.8917	1.4462	1.7415	1.9237	2.5269
3.00	.9011	1.4783	1.8000	2.0067	2.7769
3.25	.9070	1.4994	1.8408	2.0680	3.0269
3.50	.9106	1.5129	1.8682	2.1111	3.2769
3.75	.9129	1.5214	1.8860	2.1402	3.5269
4.00	.9142	1.5266	1.8973	2.1591	3.7769
5.00	.9160	1.5338	1.9130	2.1864	4.7769
6.00	.9163	1.5347	1.9152	2.1902	5.7769
7.00	.9163	1.5349	1.9155	2.1908	6.7769
8.00	.9163	1.5349	1.9156	2.1909	7.7769
9.00	.9163	1.5349	1.9156	2.1909	8.7769
10.00	.9163	1.5349	1.9156	2.1909	9.7769
∞	.9163	1.5349	1.9156	2.1909	∞

[1] Actual $\inf P(CS|R_1) = .8033$

$P^* = .85$

$\frac{nT}{\theta}$ \ k	2	3	4	5	∞
.25	.0586	.0831	.0867	.0873	.0875
.50	.2551	.3151	.3302	.3350	.3375
.75	.4376	.5355	.5651	.5771	.5875
1.00	.6028	.7441	.7907	.8118	.8375
1.25	.7478	.9394	1.0060	1.0381	1.0875
1.50	.8702	1.1189	1.2095	1.2547	1.3375
1.75	.9687	1.2799	1.3988	1.4598	1.5875
2.00	1.0440	1.4196	1.5714	1.6512	1.8375
2.25	1.0986	1.5360	1.7243	1.8263	2.0875
2.50	1.1363	1.6285	1.8551	1.9822	2.3375
2.75	1.1614	1.6982	1.9621	2.1162	2.5875
3.00	1.1775	1.7481	2.0454	2.2267	2.8375
3.25	1.1877	1.7823	2.1070	2.3133	3.0875
3.50	1.1940	1.8048	2.1503	2.3778	3.3375
3.75	1.1979	1.8192	2.1795	2.4236	3.5875
4.00	1.2003	1.8283	2.1985	2.4545	3.8375
5.00	1.2035	1.8409	2.2258	2.5010	4.8375
6.00	1.2039	1.8426	2.2297	2.5079	5.8375
7.00	1.2040	1.8429	2.2302	2.5088	6.8375
8.00	1.2040	1.8429	2.2303	2.5089	7.8375
9.00	1.2040	1.8429	2.2303	2.5089	8.8375
10.00	1.2040	1.8429	2.2303	2.5090	9.8375
∞	1.2040	1.8429	2.2303	2.5090	∞

$P^* = .90$

$\frac{nT}{\theta}$ \ k	2	3	4	5	∞
.25	.1219	.1410	.1440	.1445	.1446
.50	.3359	.3785	.3892	.3927	.3946
.75	.5399	.6085	.6289	.6372	.6446
1.00	.7314	.8306	.8624	.8768	.8946
1.25	.9077	1.0437	1.0892	1.1109	1.1446
1.50	1.0656	1.2457	1.3079	1.3385	1.3946
1.75	1.2023	1.4342	1.5169	1.5584	1.6446
2.00	1.3158	1.6064	1.7140	1.7689	1.8946
2.25	1.4053	1.7594	1.8966	1.9679	2.1446
2.50	1.4724	1.8905	2.0617	2.1528	2.3946
2.75	1.5201	1.9989	2.2065	2.3206	2.6446
3.00	1.5525	2.0816	2.3285	2.4684	2.8946
3.25	1.5738	2.1435	2.4266	2.5938	3.1446
3.50	1.5874	2.1871	2.5016	2.6955	3.3946
3.75	1.5959	2.2165	2.5560	2.7737	3.6446
4.00	1.6012	2.2356	2.5935	2.8309	3.8946
5.00	1.6083	2.2629	2.6512	2.9254	4.9846
6.00	1.6093	2.2668	2.6598	2.9406	5.8946
7.00	1.6094	2.2674	2.6610	2.9427	6.8946
8.00	1.6094	2.2674	2.6612	2.9430	7.8946
9.00	1.6094	2.2674	2.6612	2.9430	8.8946
10.00	1.6094	2.2674	2.6612	2.9430	9.8946
∞	1.6094	2.2674	2.6612	2.9430	∞

$P^* = .95$

$\frac{nT}{\theta}$ \ k	2	3	4	5	∞
.25	.1858	.1965	.1983	.1986	.1987
.50	.4177	.4401	.4457	.4476	.4487
.75	.6443	.6800	.6904	.6947	.6987
1.00	.8645	.9160	.9322	.9395	.9487
1.25	1.0764	1.1474	1.1705	1.1815	1.1987
1.50	1.2777	1.3731	1.4049	1.4203	1.4487
1.75	1.4661	1.5918	1.6342	1.6552	1.6987
2.00	1.6385	1.8015	1.8575	1.8854	1.9487
2.25	1.7918	2.0000	2.0730	2.1097	2.1987
2.50	1.9234	2.1850	2.2787	2.3264	2.4487
2.75	2.0313	2.3530	2.4724	2.5338	2.6987
3.00	2.1156	2.5013	2.6512	2.7294	2.9487
3.25	2.1780	2.6273	2.8120	2.9105	3.1987
3.50	2.2218	2.7295	2.9520	3.0742	3.4487
3.75	2.2514	2.8082	3.0689	3.2174	3.6987
4.00	2.2706	2.8658	3.1620	3.3378	3.9487
5.00	2.2981	2.9610	3.3400	3.5977	4.9487
6.00	2.3020	2.9762	3.3729	3.6536	5.9487
7.00	2.3025	2.9783	3.3777	3.6619	6.9487
8.00	2.3026	2.9786	3.3783	3.6631	7.9487
9.00	2.3026	2.9786	3.3784	3.6632	8.9487
10.00	2.3026	2.9786	3.3784	3.6632	9.9487
∞	2.3026	2.9786	3.3784	3.6632	∞

$P^* = .99$

$\frac{nT}{\theta}$ \ k	2	3	4	5	∞
.25	.2372	.2394	.2398	.2399	.2399
.50	.4835	.4881	.4893	.4897	.4899
.75	.7288	.7361	.7382	.7391	.7399
1.00	.9728	.9833	.9866	.9881	.9899
1.25	1.2151	1.2296	1.2343	1.2365	1.2399
1.50	1.4552	1.4747	1.4811	1.4842	1.4899
1.75	1.6925	1.7184	1.7270	1.7312	1.7399
2.00	1.9262	1.9603	1.9717	1.9773	1.9899
2.25	2.1553	2.1999	2.2147	2.2221	2.2399
2.50	2.3785	2.4364	2.4558	2.4655	2.4899
2.75	2.5942	2.6692	2.6943	2.7069	2.7399
3.00	2.8005	2.8970	2.9295	2.9458	2.9899
3.25	2.9949	3.1187	3.1606	3.1816	3.2399
3.50	3.1746	3.3324	3.3862	3.4133	3.4899
3.75	3.3367	3.5362	3.6051	3.6399	3.7399
4.00	3.4780	3.7275	3.8153	3.8599	3.9899
5.00	3.8141	4.3101	4.5195	4.6326	4.9899
6.00	3.8970	4.5454	4.8902	5.1078	5.9899
7.00	3.9100	4.5936	4.9880	5.2616	6.9899
8.00	3.9117	4.6007	5.0036	5.2888	7.9899
9.00	3.9120	4.6017	5.0058	5.2927	8.9899
10.00	3.9120	4.6017	5.0061	5.2932	9.9899
∞	3.9120	4.6018	5.0062	5.2933	∞

References

Alam, K. (1970). A monotonicity property of the distribution of the studentized smallest chi-square. Ann. Math. Statist., 41, 318-320.

Bain, L.F. (1978). Statistical analysis of reliability and life-testing models. Statistics: Textbook and Monographs. Vol. 24, Marcel Dekker, N.Y.

Barlow, R.E. and Proschan, F. (1966). Inequalities for linear combinations of order statistics from restricted families. Ann. Math. Statist., 37, 1574-1596.

Barlow, R.E. and Proschan, F. (1967). Exponential life test procedures when the distribution has monotone failure rate. J. Amer. Statist. Assoc., 62, 548-560.

Bechhofer, R.E. (1954). A single sample multiple decision procedure for ranking means of normal populations with known variance. Ann. Math. Statist., 25, 16-39.

Bechhofer, R.E. and Sobel, M. (1954). A single sample multiple decision procedures for ranking variances of normal populations. Ann. Math. Statist., 25, 273-289.

Berger, R.L. (1979). Minimax subset selection for loss measured by subset size. Ann. Statist., 7, 1333-1338.

Epstein, B. and Sobel, M. (1953). Life-Testing. J. Amer. Statist. Assoc., 48, 486-502.

Gibbons, J.D., Olkin, I., and Sobel, M. (1977). Selecting and Ordering Populations. John Wiley and Sons, New York.

Gupta, S.S. (1956). On a decision rule for a problem in ranking means. Mimeo. Series No. 150, Inst. of Statist., Univ. of North Carolina, Chapel Hill, North Carolina.

Gupta, S.S. (1963). On a selection and ranking procedure for gamma populations. Ann. Inst. Statist. Math., 14, 199-216.

Gupta, S.S. (1965). On some multiple decision (selection and ranking) rules. Technometrics, 7, 225-245.

Huang, W.T. and Huang, K.C. (1980). Subset selections of exponential populations based on censored data. _Proceedings of conference on recent development in statistical methods and applications_. Academia Sinica, Taipei, Taiwan.

Lehmann, E.L. (1959). _Testing Statistical Hypothesis_. John Wiley and Sons, New York.

Mann, N.R., Schafer, R.E. and Singpurwalla, N.D. (1974). _Methods for Statistical Analysis of Reliability and Life Data_. John Wiley and Sons, New York.

Ofosu, J.B. (1974). On selection of procedures for exponential distributions. _Bull. Math. Statist._, _16_, 1 - 9.

SECURITY CLASSIFICATION OF THIS PAGE

REPORT DOCUMENTATION PAGE

1. REPORT NUMBER FSU Statistics M628 USARO D52	2. GOVT ACCESSION NO. AD-A115 821	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and subtitle) Ranking and Subset Selection Procedures for Exponential Populations with Type-I and Type-II Censored Data		5. TYPE OF REPORT & PERIOD COVERED Technical 6. PERFORMING ORG. REPORT NUMBER
7. AUTHOR(s) Roger L. Berger and Jee Soo Kim		8. CONTRACT OR GRANT NUMBER(s) USARO Grant DAAG29-79-C-0158
9. PERFORMING ORGANIZATION NAME AND ADDRESS Department of Statistics Florida State University Tallahassee, Florida 32306		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS
11. CONTROLLING OFFICE NAME AND ADDRESS U.S. Army Research Office--Durham P.O. Box 12211 Research Triangle Park, NC 27709		12. REPORT DATE May, 1982 13. NUMBER OF PAGES 29
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report) 15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this report)
Distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from report

18. SUPPLEMENTARY NOTES

THE VIEW, OPINIONS, AND/OR FINDINGS CONTAINED IN THIS REPORT ARE THOSE OF THE AUTHOR(S) AND SHOULD NOT BE CONSTRUED AS AN OFFICIAL DEPARTMENT OF THE ARMY POSITION, POLICY, OR DECISION, UNLESS SO DESIGNATED BY OTHER DOCUMENTATION.

19. KEY WORDS

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

In this paper ranking and subset selection procedures for exponential populations with respect to the largest location and scale parameters are proposed. The data are assumed to be generated from Type-I and Type-II censoring mechanisms. The selection procedure proposed for the largest scale parameter based on Type-II censored data is equivalent to Gupta's procedure for gamma populations. Two procedures proposed for the selection of the largest location parameter under Type-I censoring and Type-II censoring are asymptotically equivalent.

7-8